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STATISTICAL METHODS OF PROBABLE USE
FOR UNDERSTANDING REMOTE SENSING DATA

by

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This report outlines several new statistical approaches to data problems likely to be encountered when remote sensing methods are used. The methods described are robust regression, smoothing, and modeling and estimation of ice pressure ridge characteristics.

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STATISTICAL METHODS OF PROBABLE USE FOR UNDERSTANDING REMOTE SENSING DATA

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1. INTRODUCTION

Statistical methodology has long been a familiar tool for use in understanding our natural environment. Classical examples of applications of statistics are seen in weather forecasting, in evaluation of attempts at weather modification by cloud seeding, and in descriptions of the fluctuations in the sea surface. Now the accessibility of new and extensive data from a variety of remote sensing sources, such as earth orbiting and geostationary satellites, again calls for the development and application of appropriate statistical methodology. Classical methods of statistics and of probability modeling frequently must be adapted to the new needs. The process of adaptation will proceed most efficiently if statisticians work cooperatively with the scientists actually obtaining data and studying the associated natural phenomena. Conferences such as PRIMARS I are of great value in promoting the necessary interchange of information and the stimulus to approach novel and difficult problems in a realistic manner.

This paper describes new approaches to the analysis of data, in particular to quite "noisy" data of the sort that is likely to be encountered when observing the natural environment. The descriptions given will necessarily be brief, but an attempt will be made to show how the methods and viewpoints presented may be applied to problems arising in remote sensing.

2. ROBUST METHODOLOGY: REQUIREMENTS AND POSSIBILITIES

Many scientists who have closely examined real data have encountered occasional, or even frequent, anomalous behavior. Apparent anomalies in data may be with respect to either

(a) preconceptions as to "proper" data behavior, these perhaps being buttressed by (physical) theory, <u>or</u>

(b) the nature of the general pattern of the data, especially those data points in the immediate neighborhood, e.g., in time or space.

2.1. <u>Plots</u>

In simple circumstances graphical plots will quickly reveal those points that are blatant anomalies. For instance suppose that one wishes to investigate data concerning the relationship between wind velocity and whitecap cover in the ocean. Theory may suggest a specific relationship, e.g. that white-cap cover, C, be nearly a cubic function of wind velocity v, so that it will be tempting to plot C <u>vs</u> v and note an appearance as shown on Fig. 1; there solid black dots represent (simulated) raw data. Since the eye finds it difficult to distinguish curves of the form $C = \alpha v^2$, $C = \alpha v^3$, $C = \alpha v^{7/2}$, etc., from one another, and yet is sensitive to departures from linearity, a graph of C <u>vs</u> v^3 suggests itself, but is not included here. A plot on log-log paper may be still better.

As presented, the data conforms in general to the theorized relationship or scaling, with the obvious exception of the circled point to the right. Such an anomalous point, or points, represents a challenge both to statistical technology and to the ultimate user of the data. Statistical technology assumes the responsibility for revealing the presence of such points, and, if possible, for providing a meaningful and useful summary of the remaining points. It falls to the consumer or ultimate user of the data, preferably with the help of a subject-matter specialist (physicist or oceanographer) to interpret the apparently anomalous maverick--or exotic, or outlying--data point: is it

(i) an evidence of the failure of the relation $C = \alpha v^3$, say for large velocities,

or is it

(ii) an outright error in data recording, and to be disregarded?

being just two possible options.

Note that simple graphs are invaluable for pointing out extreme outliers in simple, one explanatory variable, situations. If more variables are required, informative plots are more difficult without the use of more statistical technology. We next show that classical, least-squares, technology may be quite misleading, but that replacements are available. See Mosteller and Tukey (1977), abbreviated MT hereafter.

2.2. Fits and Residual Plots

Suppose that one wishes to summarize data such as that in Fig. 1 by fitting the relationship $C = \alpha v^3$, i.e., determining the parameter α from the data. The classical and automatic way of doing so is to apply least squares; computer programs are universally available, even for handheld calculators (the TI 59, or HP 67). What are we likely to find? A least-squares line (treating $w = v^3$ as the independent variable presents $C = \alpha w$; one can also plot and fit $C^{1/3} = (a)^{1/3}v$, and there may be reasons for this choice) is quite apt to fatally misrepresent the situation, responding much too sensitively to the single (here encircled) outlying value, and straying systematically away from the main body of the data; see the points represented by o in Fig. 1.

An alternative method for fitting, described in MT, is less susceptible to outlier influence--is far more robust to departures from basic assumptions--than is the ordinary least squares (OLS) method. This new method, termed biweight fitting, is carried out by a procedure that uses the OLS computation iteratively. In the course of the computations weights are automatically developed that reduce the influence of the encircled value of Fig. 1, permitting the fit to more closely approximate the main body of the data. We now describe and illustrate the biweight fitting procedure as it is adapted to the problem of determining the parameter α in the relation y_i vs αx_i.

Biweight Fitting Calculation

(1) Compute the kth ($k = 1,2,3,\ldots$) iterative estimate of α, denoted by $\alpha^{(k)}$ by solving

$$\sum_{i=1}^{n} (y_i - \alpha^{(k)} x_i) x_i w_i^{(k-1)} = 0 \ ,$$

to obtain

$$\alpha^{(k)} = \frac{\sum_{i=1}^{n} y_i x_i w_i^{(k-1)}}{\sum_{i=1}^{n} x_i^2 w_i^{(k-1)}} \ ,$$

(2) the weights, $w_i^{(k-1)}$, are of this form:

$$w_i^{(k-1)} = \left[1 - \left(\frac{y_i - \alpha^{(k-1)} x_i}{cS^{(k-1)}} \right)^2 \right] \qquad \text{if } (\cdot) \leq 1$$

$$= 0 \qquad \text{if } (\cdot) > 1$$

where $(\cdot)$ refers to the term $[(y_i - \alpha^{(k-1)} x_i)/S^{(k-1)}]$; $S^{(k-1)}$ is a scale factor (robust replacement for the standard deviation) that may be computed in the following manner.

(3) The $k\text{-}1^{st}$ iterated value of the scale factor is

$$S^{(k-1)} = \text{median}\{|y_i - \alpha^{(k-1)} x_i|\},$$

c being a constant of value 6, or 9;

(4) the first value, $\alpha^{(1)}$, of the iterative sequence can be obtained by equalizing all weights $(w_i^{(0)} = 1)$, which is equivalent to OLS; alternatively, one can utilize a "robust start," suggestions for which can be found in MT.

The iteration is carried on until the difference between successive values is small; usually 4 to 8 iterations is sufficient. The resulting α-estimate can be denoted by $\hat{\alpha}$.

Following the fitting it is informative to plot the residual values:

$$r_i = y_i - \hat{\alpha}x_i \equiv y_i - \hat{y}_i \,, \qquad i = 1,2,\ldots,n,$$

$\hat{y}_i$ being shorthand for the predicted y value. In case there is a single outlier, as in Fig. 1, the fitted line will tend to hug the major point cloud, and a histogram of the residuals will dramatically reveal the presence of the outlier, suggesting further investigation. A plot of r_i vs y_i is also useful. See MT for further suggestions.

2.3. Numerical Illustration

The following are a set of (simulated) whitecap percentages and corresponding wind velocities. Alongside are values for white cap coverage estimated by OLS and by the biweight procedure.

Velocity	Cover ("Actual")	Cover (OLS Estimate)	Cover (Robust Estimate)
2	0.011	0.011	0.0067
7	0.63	0.49	0.29
10	1.30	1.43	0.84
15	3.89	4.82	2.83
18	2.89	8.33	4.88
21	8.16	13.2	7.76
24	25.7	19.7	11.6

It is clear from the above table, and perhaps clearer from Fig. 1, that the OLS solution, in its attempt to fit the point $C(24) = 25$, systematically and considerably over-estimates the points at $v = 15$ and above. The biweight estimator performs much better, allowing a closer fit to all data other than $C(24)$. A residual plot brings attention to bear on that point.

Since the values of "Actual Cover" were actually constructed by forming $0.008v^3$ and adding Gaussian random noise with value proportional to $C(v)$, and since the sequence of values of $0.008v^3$ were 0.0064, 0.27, 0.80, 2.7, 4.67, 7.41, 11.06, we cannot fault the manner in which the biweight procedure functioned in this example and are encouraged to use it more widely.

2.4. Possible Application to Remote Sensing Data

In a paper in this conference proceedings by Depriest (1979), and in Fleming (1979), a problem arising from partial cloud cover contamination of remote sensing data is described and addressed. This problem has the following origin. A series of measurements are made on a physical quantity (sea surface temperatures) but are contaminated. That is, in the case of sea surface temperatures, if no clouds are present the measurements are approximately normally distributed around μ (the true temperature). However, if clouds are present a fraction of the measurements are made artificially smaller, cloud temperatures being lower than those at earth surface. The problem is to estimate μ. Techniques for doing so are described by Depriest (1979) and by Fleming (1979). We describe a possible alternative approach that uses robust regression. Operational characteristics of the two procedures have not yet been compared.

(1) Arrange the measurements in order: $y_1 < y_2 < y_3 < \cdots < y_{n-1} < y_n$. The largest observations may well appear similar to the largest order statistics of a normal distribution with (unknown) mean μ and standard deviation σ (sometimes assumed known, although caution is in order), while the smaller ones are likely to depart systematically.

(2) Carry out a preliminary plot of

$$y_k \ \underline{\text{vs}}\ \Phi^{-1}\left(\frac{k}{n+1}\right), \qquad k = n, n-1, n-2, \ldots$$

where $\Phi^{-1}(p)$ is the inverse function of the unit normal: recall that if

$$\Phi(y) = \int_{-\infty}^{y} \exp(-\tfrac{1}{2} z^2) \frac{dz}{\sqrt{2\pi}} ,$$

is the unit normal distribution, then the solution of the equation $\Phi(y) = p$ gives

$$y(p) = \Phi^{-1}(p);$$

Φ, and hence Φ^{-1}, are widely tabulated. Alternatively, use Arithmetic Probability Paper. If y_k is an ordered observation from a normal population, then the plot should appear straight, while a systematic departure from linearity indicates a departure from normality. Suppose departures begin to occur at $k = D$; sometimes D may be greater than $n/2$. One may first eye-fit a straight line to the points $k = n, n-1, \ldots , D$. Then $\hat{y}_{n/2} = \hat{\mu}$ (estimated temperature). i.e. the value of the fitted line at $n/2$ should give a reasonable value for μ.

(3) Going further in a formal direction, one may wish to fit a line to the data points. Here a biweight fit should behave well, tending to be oblivious to spurious (cloud contamination) points. One can proceed to fit the relation

$$y_k \ \underline{\text{vs}} \ \mu + \sigma x_k$$

with

$$x_k = \Phi^{-1}\left(\frac{k}{n+1}\right) , \qquad k = n, n-1, n-2, \ldots ;$$

a start using the eye-fit to points $k = n, n-1, \ldots, D$ may be worthwhile. Finally, quote the estimate

$$\hat{\mu} = \text{med } \hat{y}_k = \frac{1}{2}(\hat{y}_{n/2} + \hat{y}_{(n/2)+1}), \qquad n \text{ even};$$

$$= \hat{y}_{(n+1)/2}, \qquad n \text{ odd}.$$

where

$$\hat{y}_k = \hat{\mu} + \hat{\sigma} x_k .$$

The above procedure seems worth further investigation and refinement. One important step may be to adjust for the effect of correlation between order statistics when carrying out the regression.

3. SMOOTHING DATA

If one plots certain environmental data, e.g. monthly total rainfall, or perhaps daily maximum temperature, at a particular location, systematic regularities seem to appear, but may be masked by noise. Often there is a seasonal pattern, i.e. one that is roughly cyclic in nature. Attempts to fit such a pattern with polynomials is doomed to failure, and selection of a set of sines and cosines that does well (Fourier series) may lead to many terms. Some method of smoothing the original series that lays bare the regularities is to be desired. After such is made available, one can study the residuals around it. Spectral analysis or some such formal procedure may then be of use.

Classical smoothing procedures involve some form of moving average, and are susceptible to the python-swallowing-the pig difficulty: imagine using the linear smoothing operation

$$sy_t = \frac{y_{t-1} + y_t + y_{t+1}}{3}$$

on the y_t series

t	1	2	3	4	5	6	7	8	9	10
y_t	11	9	7	8	8	29	10	8	6	9
Sy_t	11	9	8	7.7	15	15.7	15.7	8	7.7	10
Ry_t	11	9	8	8	8	10	10	8	8	10

clearly the entry of 29 at $t = 5$ into the smoothed value series Sy_t gives a serious distortion, travelling as it does in partially digested form through the next two terms of the smoothed series. If further smoothing is attempted the bulge is reduced slightly, but spreads out in time.

On the other hand, the non-linear operation of taking running medians, as suggested by Tukey (1977), performs effectively (in both cases circled and values are copied from the original series; more sophisticated procedures can be invented as well). The last row in the table, labelled Ry_t, gives the result of this robust smoothing; note that it behaves in an intuitively appealing manner, essentially ignoring the outlying value 29. Further steps can be taken to improve the "smooths," but we refer to Tukey (1977) Chapters 7 and 16 for details.

Two further points may be made. The first is that the analysis of a sequence of data points, and their projection or forecasting in space or time, should not end with providing a smoothed or averaged version. Examination of the remaining variation, e.g. the sequence $y_t - Ry_t$, called the "rough" by Tukey may well be rewarding; presence and suggestiveness of various outliers is much more evident in the rough (residuals) sequence than in the original sequence of data points. Secondly, the procedure described for smoothing simple sequences of data points must be adapted to planar (two-dimensional) data; some work has been done, but much remains.

4. STOCHASTIC MODELING OF ICE PRESSURE RIDGES

The dynamics of ice formation in the Earth's cold regions results in the development of irregular ice pile-ups, or pressure ridges. These ridges occur in an apparently random fashion in space; in fact the following regularities are observed by remote sensing methods (courtesy of Dr. W. Weeks, in a seminar at the Naval Postgraduate School, Monterey, California, Winter, 1979; see also Weeks, et al. (1979)):

1) Along a sampling line (e.g. airplane flight path, or straight submarine track) ice ridges seem to appear in accordance with a stationary Poisson process, so if $R(x)$ is the number of such ridges encountered over a distance x, then approximately

$$P\{R(x) = n\} = e^{-\lambda x} \frac{(\lambda x)^n}{n!}, \qquad n = 0,1,2,...$$

where $\lambda > 0$ is the density of ice ridges.

2) The probability distribution of ridge "sail heights" (or "keel depths") may be approximated by the forms $F(y) = 1-e^{-\mu y}$, or $1 - e^{-\nu y^2}$; the best-fitting distribution may well depend upon the method of observation (averaging properties).

For further details see work referenced in Weeks <u>et</u> <u>al</u>. (1979).

Now it may be of interest to compute the distribution of the <u>maximum</u> sail height, or keel depth, that one is to encounter over a course of length x. This is very simple, <u>given</u> the particular distributions of sail number and size <u>and</u> furthermore assuming independent between ridge heights. Let $\bar{H}(x)$ be the maximum sail height; then

$$P\{\bar{H}(x) \leq y\} = \sum_{n=0}^{\infty} e^{-\lambda x} \frac{(\lambda x)^n}{n!} [F(y)]^n$$

since all of the Poisson-distributed heights must be less than y in order for the maximum to be below y.

Sum out to obtain

$$P\{\bar{H}(x) \leq y\} = \exp\{-\lambda x[1-F(y)]\}$$

Depending upon which distribution is picked for ridge heights, we get

a) $P\{\bar{H}(x) \leq y\} = \exp(-\lambda x e^{-\mu y})$

b) $P\{\bar{H}(x) \leq y\} = \exp(-\lambda x e^{-\nu y^2})$

These closely resemble classical extreme value distributions. Note that if logs are taken simplicity occurs:

a') $\ln P\{\bar{H}(x) \leq y\} = -\lambda x e^{-\mu y}$;

$\ln(-\ln P\{\bar{H}(x) \leq y\}) = \ln(\lambda x) - \mu y$

b') $\ln P\{\bar{H}(x) \leq y\} = -\lambda x e^{-\nu y^2}$;

$\ln(-\ln P\{\bar{H}(x) \leq y\}) = \ln(\lambda x) - \nu y^2$.

If either of these formulas are to be used for practical purposes, values of the parameters must be obtained. In order to estimate parameters λ, μ, ν in the above models from data one naturally thinks of the method of maximum likelihood. Suppose that we have observed $R(x) = n$ ridges of heights $y_1, y_2, \cdots, y_n$. Then the maximum likelihood estimates are

$$\hat{\lambda} = \frac{x}{n}, \quad \hat{\mu} = \frac{1}{\bar{y}}, \quad \hat{\nu} = \frac{1}{\overline{y^2}}$$

where as usual we have put

$$\overline{y^k} = \frac{1}{n} \sum_{i=1}^{n} y_i^k$$

Hence our estimates are of the form

a") $\text{est } \ell n(-\ell n\ P\{\bar{H}(x) \leq y\}) = \ell n\ \hat{\lambda} + \ell n\ x - \hat{\mu} y$

b") $\text{est } \ell n(-\ell n\ P\{\bar{H}(x) \leq y\}) = \ell n\ \hat{\lambda} + \ell n\ x - \hat{\nu} y^2$

If rather large samples are available <u>and</u> if distributional assumptions are well satisfied one may feel comfortable with conventional standard errors based on Fisher information and normality; see Cramér (1946). On the other hand, it is of interest to apply the <u>jackknife technique</u> (see R. G. Miller (1974) for a review) to obtain estimates of the variance of estimate due particularly to the ridge heights. To carry out the calculation, (i) compute $\hat{\nu}_{all} = n/\overline{y^2}$; then (ii) compute

$$\hat{\nu}_{(-j)} = \frac{n-1}{y_1^2 + y_2^2 + \cdots + y_{j-1}^2 + 0 + y_{j+1}^2 + \cdots + y_n^2}$$

for $j = 1,2,\ldots, n$; then (iii) compute the pseudovalues $\hat{\nu}_j = n\hat{\nu}_{all} - (n-1)\hat{\nu}_{(-j)}$, and (iv) average to obtain a jackknifed point estimate $\hat{\nu}_{JK} = (1/n)\sum_{j=1}^{n}\hat{\nu}_j$, and its variance

$$\frac{1}{r}\left[\frac{1}{n-1}\sum_{j=1}^{n}(\hat{\nu}_j - \hat{\nu}_{JK})^2\right] = s^2_{\hat{\nu}_{JK}} .$$

Then we can estimate the standard error of the probability predictior e.g. b") by computing

$$\text{S.E.} = (\text{Var}[\text{est } \ln(-\ln P\{\bar{H}(x) \leq y\})])^{1/2}$$

$$\simeq \left(\frac{1}{\hat{\lambda}_{JK}} s^2_{\hat{\nu}_{JK}} + y^2\, s^2_{\hat{\nu}_{JK}}\right)^{1/2}$$

A similar calculation is easily performed for model a); details are omitted. From the above results, approximate confidence intervals may be constructed for the probability of encountering a (maximum) ridge sail height less than y in magnitude.

Fairly recent theoretical results of Efron and Hinkley (1978) suggest that if a traditional maximum likelihood approach is taken, one is better off using observed Fisher information rather than expected Fisher information in order to establish an approximate standard error in either case a) or b). However, work of Reeds (1978) suggests that use of the jackknife in conjunction with maximum likelihood yields results that tend to be rather independent of the basic model chosen. Both of these suggestions must be validated by further work, a good deal of which will necessarily involve Monte Carlo simulation. Such work should be of great importance and interest to those who must assess the probabilities of extreme, rare, events, and who furthermore wish to provide some reasonably valid estimates of the error of their estimates.

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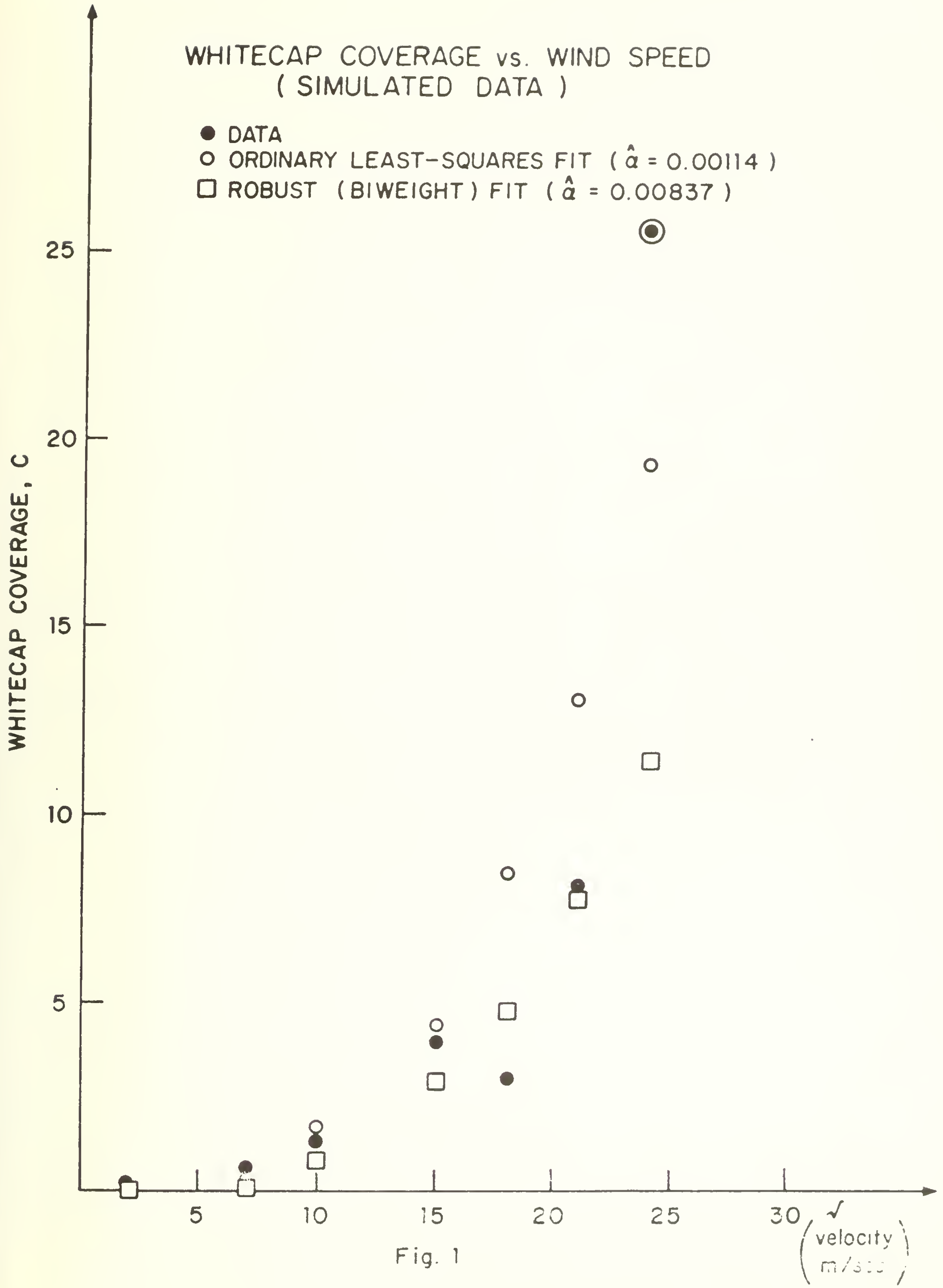
WHITECAP COVERAGE vs. WIND SPEED
(SIMULATED DATA)
DATA
ORDINARY LEAST-SQUARES FIT ($\hat{\alpha}$ = 0.00114)
ROBUST (BIWEIGHT) FIT ($\hat{\alpha}$ = 0.00837)
WHITECAP COVERAGE, C
25
20
15
10
5
5
10
15
20
25
30
velocity

Fig. 1

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